Pergamon

Computers chem. Engng Vol. 20, Suppl., pp. S931–S936, 1996

S0098-1354(96)00163-9
0098-1354/96 $15.00+0.00

FUZZY PREDICTIVE CONTROL BASED ON RELATIONAL MATRIX MODELS

Igor Škrjanc, Katarina Kavšek-Biasizzo, Drago Matko
Faculty of Electrical and Computer Engineering
University of Ljubljana, Slovenia
Tržaška 25, 61000 Ljubljana
e-mail: *igor.skrjanc@fer.uni-lj.si*

Oliver Hecker
Technical University of Darmstadt
Institute of Automatic Control
Landgraf-Georg Str.4, 64283 Darmstadt, Germany

ABSTRACT

The paper represents a new aproach to the predictive model-reference control. The prediction of the process output signal is made on the basis of fuzzy process model. Using the fuzzy model of the process the forecast of the process output over a certain horizont in the future is calculated and can be used as a predictor in the long-range predictive control strategy. The concept is implemented on real industrial scale temperature plant.

KEYWORDS

predictive control, fuzzy relational model, identification, nonlinear systems

1.INTRODUCTION

The predictive control has become a very important area of research in recent years. The principal is based on the forecast of the output signal y at each sampling instant. The forecast is made implicitly or explicitly according to the model of the process to be controlled. In the next step the control is selected which brings the predicted process output signal back to the reference signal in way to minimize the area between the reference and the output signal. The fundamental methods which are essentially based on the principal of predictive control are Richalet's method (Richalet et al., 1976, Model Algorithmic Control), Cutler's method (Dynamic Matrix Control), De Keyser's method (Extended Prediction Self-Adaptive Control) and Ydstie's method (Extended Horizon Adaptive Control).

According to the process model two main approaches have been developed in the area of predictive control. The first one is based on parametric model of the controlled process. The parametric model could be described in form of transfer-function or in state-space domain. An important disadvantage of using the parametric model is that it represents a linearized model of the process. The control of the strong nonlinear processes could be unsatisfactorilly. The second approach proposed in literature is based on nonparametric model. The advantage of this approach is that the model coefficient can be obtained directly from samples of the input and output responses without assuming the model structure. In our example a scheme of predictive control based on fuzzy relational matrix model is proposed, which represents a combination of nonparametric and parametric approach to the predictive control.

Predictive control based on fuzzy relational matrix model is capable to control also very difficult processes, such as nonlinear processes, processes with long time delay and non-minimum phase. The controllers based on prediction strategy also exhibit remarkable robustness with respect to model mismatch and unmodeled dynamics.

The first part of the paper deals with the concept of fuzzy relational matrix modelling. In the second part the concept of fuzzy predictive control is given. Finally, the implementation of fuzzy predictive control on the real temperature plant is presented.

2.TEMPERATURE PLANT

The heart of the temperature plant is a tubular heat exchanger, through which steam from an electrically heated steam generator continuously circulates in counter-current flow to water circuit. A schematic diagram is shown in Figure 1. Temperature of the steam is kept constant by a local presure control in the steam generator and the flow of the steam is controlled by position of the steam valve. After heating in the exchanger, the water passes through a pneumatic valve into the air cooler and then reenters the exchanger.

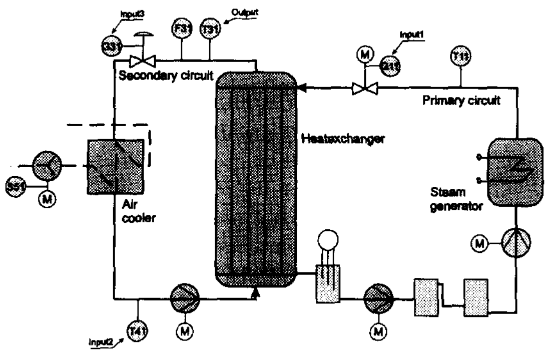

Figure 1: Temperature plant

The output of the whole process is the temperature of the water leaving the exchanger (T31). The output is controlled by the position of the steam valve (G11) in the primary circuit which represents an input signal.

Behaviour of the process strongly depends on operating conditions, which are defined by other signals applied to the process: temperature at the outlet of the air cooler (T41) and pneumatic valve position (G31). With approximatelly constant temperature T41, the setting of the pneumatic valve position results in significant changes of the process gain.

3.FUZZY IDENTIFICATION

Fuzzy logic appears to be a very promising approach in process automation, specially fuzzy modelling and fuzzy control. Fuzzy modelling or identification means to find a set of fuzzy if-then rules with well defined attributes, that can describe the given I/O behaviour of the process. In the recent years many different approaches for fuzzy identification have been proposed in the literature, by Tong [5], Pedrycz and Czogala [3], [4], Sugeno [1], [2].

The fuzzy identification algorithm used in this paper is based on fuzzy relational matrix model with crisp output variable.

3.1.THE IDENTIFICATION ALGORITHM

Suppose the rule base of a fuzzy system is as follows:

$$\boldsymbol{R}_i : \textbf{IF } x_1 \textit{ is } \boldsymbol{A}_i \textit{ and } x_2 \textit{ is } \boldsymbol{B}_i \textbf{ THEN } y = r_i$$

$$i = 1, \dots N \tag{1}$$

where x_1 and x_2 are input variables of the process, y is an output variable, A_i, B_i are fuzzy sets characterized by their membership functions and r_i are the crisp values. Such a very simplified fuzzy model can be regarded as a collection of several linear models applied locally in the fuzzy regions, defined by the rule premises. The idea behind this kind of modeling is close to well-known concept of gain scedulling.

Rule-premises are formulated as fuzzy AND relations on the cartesian product set $X = X_1 \times X_2$, and several rules are connected by logical OR. Fuzzification of a crisp value x_1 produces a column vector

$$\boldsymbol{\mu}(x_1) = [\mu_{A_1}(x_1), \mu_{A_2}(x_1), \dots \mu_{A_m}(x_1)]^T \tag{2}$$

and similarly for a crisp value x_2. The degrees of fulfillment of all possible AND combinations of rule premises are calculated and written into matrix $\boldsymbol{S}$. If the algebraic product is used as AND operator, this matrix can be directly obtained by multiplication:

$$\boldsymbol{S} = \boldsymbol{\mu}_1 \otimes \boldsymbol{\mu}_2^T = \boldsymbol{\mu}_1 \cdot \boldsymbol{\mu}_2^T. \tag{3}$$

A crisp output value y is computed by simplified algorithm for singletons as a weighted mean value (Center of Singeltons):

$$y = \frac{\sum_{i=1}^{n} \sum_{j=1}^{m} s_{ij} r_{ij}}{\sum_{i=1}^{n} \sum_{j=1}^{m} s_{ij}}. \tag{4}$$

The dimension of matrix $\boldsymbol{S}(m \times n)$, which actually represents the structure of the model, depends on the dimensions of input fuzzy sets $\boldsymbol{\mu}_1(m \times 1)$ and $\boldsymbol{\mu}_2(n \times 1)$. The fuzzy relational matrix $\boldsymbol{R}$ consists of elements r_{ij}.

In order to apply a standard least-squares method to estimate the parameters r_{ij}, the vectors $\boldsymbol{s}$ and $\boldsymbol{r}$ are formed from $\boldsymbol{S}$ and $\boldsymbol{R}$, respectively:

$$\begin{aligned} \boldsymbol{s} &= (s_{11}\; s_{12} \ldots s_{1n} \ldots s_{m1}\; s_{m2} \ldots s_{mn})^T \\ \boldsymbol{r} &= (r_{11}\; r_{12} \ldots r_{1n} \ldots r_{m1}\; r_{m2} \ldots r_{mn})^T \end{aligned} \tag{5}$$

Using these vectors, equation 4 is rewritten as:

$$y = \frac{\boldsymbol{s}^T \cdot \boldsymbol{r}}{\boldsymbol{s}^T \cdot \boldsymbol{I}} \tag{6}$$

where $\boldsymbol{I}$ defines the vector of ones of the same dimension $(n \cdot m \times 1)$ as $\boldsymbol{s}$ and $\boldsymbol{r}$. The elements r_{ij} are estimated on the basis of the observations which are obtained in equidistant time intervals by measuring the process input and output. A system of linear equations is constructed from upper equations for the time intervals $t = t_1, t = t_2, \ldots, t = t_N$:

$$\begin{bmatrix} \boldsymbol{s}^T(t_1) \\ \boldsymbol{s}^T(t_2) \\ \vdots \\ \boldsymbol{s}^T(t_N) \end{bmatrix} \cdot \boldsymbol{r} = \begin{bmatrix} \boldsymbol{s}^T(t_1) \cdot \boldsymbol{I} y(t_1) \\ \boldsymbol{s}^T(t_2) \cdot \boldsymbol{I} y(t_2) \\ \vdots \\ \boldsymbol{s}^T(t_N) \cdot \boldsymbol{I} y(t_N) \end{bmatrix}. \tag{7}$$

The system is of the form:

$$\boldsymbol{\Psi} \cdot \boldsymbol{r} = \boldsymbol{\Omega} \tag{8}$$

with a known nonsquare matrix $\boldsymbol{\Psi}$ and a known vector $\boldsymbol{\Omega}$. The solution of this overdetermined system is obtained by taking the pseudo-inverse as an optimal solution of vector $\boldsymbol{r}$ in a least squares sense:

$$\boldsymbol{r} = (\boldsymbol{\Psi}^T \boldsymbol{\Psi})^{-1} \boldsymbol{\Psi}^T \boldsymbol{\Omega} \tag{9}$$

where $\boldsymbol{\Psi}$ stands for fuzzified data matrix with dimension $N \times (n \cdot m)$ and $\boldsymbol{\Omega}$ has dimension $N \times 1$.

In the case of more than two input variables (MISO multi-input-single-output fuzzy system), matrices $\boldsymbol{S}$ and $\boldsymbol{R}$ are no longer matrices, but both become a tensor, defined in the total product space of the inputs.

3.2.FUZZY MODELING OF THE TEMPERATURE PROCESS

Similarly to static feedforward neural network, fuzzy models actually represent a static mapping between model input fuzzy sets and output fuzzy sets, so dynamic systems are then modelled as a nonlinear static mapping between the fuzzy sets defined in the space of lagged model inputs and outputs. Note that in the same way the system dynamics are captured in other kinds of models like linear regression models or neural networks.

For the temperature process a MISO fuzzy model was identified. The model rule base approximates a first-order nonlinear regression model where the new temperature T31 is a function of the current temperature T31, the steam valve position G11, the current temperature T41 and the current position of the pneumatic valve G31:

$$y(k+1) = f(y(k), u_1(k), u_2(k), u_3(k), u_4(k)) \tag{10}$$

In each fuzzy input space we chose only three equally spaced triangulary shaped fuzzy sets. In generaly, the estimation of the membership functions (shape, number, position,) and determination of the rules can be solved with several methods (genetic algorithms, neural network, clustering,..).

The fuzzy model of the process is given by four-dimensional hyperspace structure $\boldsymbol{R}$.

4.FUZZY PREDICTIVE MODEL-BASED CONTROL

In this section the basics of predictive control based on convolution theorem is introduced. The convolution model is described using the following equation

$$y(k) = \sum_{i=1}^{\infty} g_i \Delta u_{k-i} + n(k) \tag{11}$$

where $y(k)$ represents the output signal, $\Delta u(k)$ is input signal and g_i are coeficients of process step response and $n(k)$ describes the unmodeled dynamics.

Control signal in the case of model-reference predictive control is obtained optimizing the criterion function according to the variable $\Delta u(k+j)$

$$J = \sum_{j=N_1}^{N_2} (\hat{y}(k+j) - y_m(k+j))^2 + \lambda \sum_{j=0}^{N_u-1} (\Delta u(k+j))^2 \tag{12}$$

where $\Delta u(k+j) = 0$ for $j \geq N_u$ is assumed. The variable $y_m(k+j)$ represents j steps ahead prediction of reference signal , $\hat{y}(k+j)$ describes the prediction of output signal obtained using the process model and $\Delta u(k+j)$ stands for prediction of control signal. The values N_1 and N_2 are minimal and maximal prediction horizon and N_u denotes prediction horizon of control signal. The parameter λ represents the weight of the control signal.

The output signal prediction can be divided on free respons of the process $y_p(k+j)$ and forced response $y_v(k+j)$ as follows

$$\hat{y}(k+j) = y_p(k+j) + y_v(k+j) + n(k) \tag{13}$$

Free response of the process denotes the behaviour where $\Delta u(k+j) = 0$ for $j = 1, \ldots, N_u$ is assumed. Forced response describes the behaviour in the case of input signal $\Delta u(k+j)$ for $j > 0$.

The output signal prediction is according to the superposition described in the following form

$$\hat{y}(k+j) = \sum_{i=1}^{j} g_i \Delta u(k+j-i) + \sum_{i=j+1}^{\infty} g_i \Delta u(k+j-i) + n(k) \tag{14}$$

or

$$\hat{y}(k+j) = y(k) + \sum_{i=1}^{j} g_i \Delta u(k+j-i) + \sum_{i=j+1}^{\infty} g_i \Delta u(k+j-i) - \sum_{i=1}^{\infty} g_i \Delta u(k-i) \tag{15}$$

Previous equation can be described also in the compact form

$$\hat{y}(k+j) = \boldsymbol{G}_j \Delta u(k+j) + p_j \tag{16}$$

where $\boldsymbol{G}_j = [g_1 g_2 \ldots g_j]$ denotes the vector of step response coefficients, p_j represents the free response of the system given by equation

$$p_j = y(k) + \sum_{i=j+1}^{\infty} g_i \Delta u(k+j-i) - \sum_{i=1}^{\infty} g_i \Delta u(k-i) \tag{17}$$

$$p_j = y(k) + \sum_{i=1}^{\infty} (g_{j+i} - g_i) \Delta u(k-i) \tag{18}$$

In the case of asimptoticaly stable processes the upper equation can be given in the following form

$$p_j = y(k) + \sum_{i=1}^{N_2} (g_{j+i} - g_i) \Delta u(k-i) \tag{19}$$

where the maximum prediction horizon N_2 is chosen to fulfill the equation

$$g_{j+i} - g_i \cong 0, \tag{20}$$

for $i > N_2$ and $j = N1, \ldots N_2$.

The equation 12 can be in the compact matrix form described as follows

$$J = (\boldsymbol{y}_p - \boldsymbol{y}_m)(\boldsymbol{y}_p - \boldsymbol{y}_m)^T + \lambda \Delta \boldsymbol{u} \Delta \boldsymbol{u}^T \tag{21}$$

where $\boldsymbol{y}_p = [\hat{y}(k + N_1), \ldots, \hat{y}(k + N_2)]^T$ denotes the vector of output signal prediction, $\boldsymbol{y}_m = [y_m(k + N_1), \ldots, y_m(k + N_2)]^T$ is vector of reference signal prediction between minimal and maximal prediction horizon. Vector $\Delta \boldsymbol{u} = [\Delta u(k) \ldots \Delta u(k + N_u - 1)]^T$ represents the sequence of control signal.

The prediction of the output signal in the compact matrix form is the following

$$\boldsymbol{y}_p = \boldsymbol{G} \Delta \boldsymbol{u} + \boldsymbol{p} \tag{22}$$

where

$$\boldsymbol{G} = \begin{bmatrix} g_{N_1} & 0 & 0 & \cdots \\ g_{N_1+1} & g_{N_1} & 0 & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ g_{N_2} & g_{N_2-1} & \cdots & g_{N_2-N_u+1} \end{bmatrix} \tag{23}$$

and $\boldsymbol{p} = [p_{N_1} \ldots p_{N_2}]$.

Considering the previous equations the criterion function can be described

$$J = (\boldsymbol{G}\Delta \boldsymbol{u} + \boldsymbol{p} - \boldsymbol{y}_m)(\boldsymbol{G}\Delta \boldsymbol{u} + \boldsymbol{p} - \boldsymbol{y}_m)^T + \lambda \Delta \boldsymbol{u} \Delta \boldsymbol{u}^T \tag{24}$$

The optimal solution of criterion function is obtained in the following form

$$\Delta \boldsymbol{u} = (\boldsymbol{G}^T \boldsymbol{G} + \lambda \boldsymbol{I})^{-1} \boldsymbol{G}^T (\boldsymbol{y}_m - \boldsymbol{p}) \tag{25}$$

The solution is given in vector form and provides the calculation of input signal for N_u values in advance where only the first value is applied to the process. In the next sampling period the solution is computed again and another set of N_u values of the control is obtained according to a receding horizon strategy.

4.1.FUZZY CONTROL USING DYNAMIC MATRIX

The main idea of fuzzy predictive control using dynamic matrix is to combine the advantages of fuzzy modeling and predictive control. The idea is based on *on-line* computing of dynamic matrix $\boldsymbol{G}$. The described method offers some advantages in the case of nonlinear processes where the dynamic depends on operating point and can be presented as $\boldsymbol{G}(u, y)$. Dynamic matrix is calculated on the basis of fuzzy process model $\boldsymbol{r}$ whenever the operating point of the system is changed. The vector $\boldsymbol{G}_j$ is calculated recursively using equation

$$g_j = \frac{\boldsymbol{s}^T(g_{j-1}, u(k) + u_{step}) \cdot \boldsymbol{r}}{\boldsymbol{s}^T(g_{j-1}, u(k) + u_{step}) \cdot \boldsymbol{I}} \tag{26}$$

for $j = 1, \cdots N_2$ where u_{step} represents the step input signal to the process. The value of g_0 is equal $y(k)$. Dynamic matrix of the system is calculated when the reference signal $y_m(k)$ is changed or the difference between process output and model reference becomes significant.

5.REAL TIME EXPERIMENT

Although the process is very complex, it could be presented as a model with dynamics, approximately described with first order with small time delay, with significantly time varying parameters and nonlinear according to the operating point. The results in the case of fuzzy predictive control for temperature plant are shown in Fig.2, where the output $y(t)$, the reference model output $y_m(t)$ and control signal $u(t)$ are presented for two different operating conditions determinated by the position of the pneumatic valve G31 (0% and 12.5% opened).

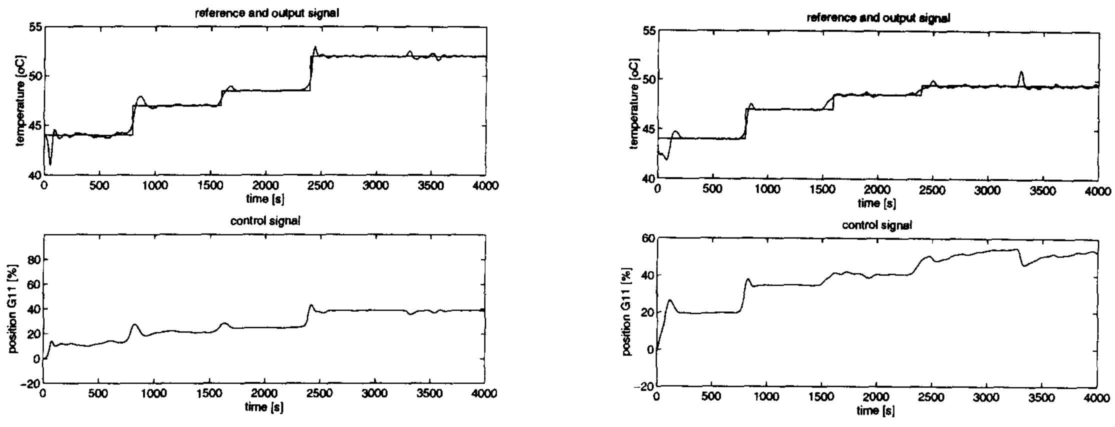

Figure 2: The process output and the reference-model signal in the case of fuzzy predictive control

6.CONCLUSION

In the paper the fuzzy predictive control algorithm is presented. Regarding to the real-time experiments on the temperature plant which exhibits a nonlinear character it can be seen that the new algorithm gives a good performance. The main advantage in comparison with other conventional techniques is use of fuzzy model which enables implementation of nonlinear processes.

ACKNOWLEDGEMENT

The work presented in this paper has been supported by the Kernforschungsanlage, Jülich, Germany, and Slovenian Ministry of Science and Technology.

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